‘Counting and doing sums is the basis of all order inside our heads.’

25 Where's My Calculator?

The glass cost 28 francs, the shop assistant said, but there was a discount of 25% on a dozen. 'That makes 21 francs a glass,' I said. The young lady looked at me in astonishment and said, 'Yes, it'll be something like that.' Then she went over to the counter, picked up the calculator, typed in 28, divided by 100, multiplied by 25 and said, almost in amazement. 'The discount is seven francs, yes, you're right, that makes 21 francs.'

Traditional arithmetic is dead and buried and has been replaced by mathematics. That means that when experienced instructors wring their hands and complain that their apprentices are hardly able to do sums, we can proudly reply, 'True, they can't do sums, but they can think.'

It has always made me furious when people suggest that the ability to do sums has nothing to do with thinking. I remember an eleven-year-old who had a very poor memory and once more couldn't remember what 7 x 8 was. His solution was as follows: 'I'll try doing 8 x 8, that's the same as 4 x 16 and that's 2 x 32, equals 64. Take away 8, that makes 56.' And if that's not thinking, I'm a Dutchman.

In the late 1960s arithmetic was subjected to a fundamental critical review and the syllabus and course books throughout Switzerland were revised in accordance with the latest trends. It is often suggested that what sparked off this total review of traditional arithmetic teaching was the 'Sputnik shock' the Americans suffered in 1957 when they realised the Russians were clearly superior to them in science and technology.

At the time there were three main arguments against the traditional teaching concentrating on arithmetic:

Firstly, it was complained that arithmetic *fixed the pupils' thought processes* in very specific channels and hampered the development of true mathematical thinking. Mathematical thinking was *flexible, general and creative*, it was said, and could not be developed by concentrating on the number system and familiarising pupils with it by means of the basic operations. The important thing was to teach pupils how to deal with abstract quantities and logical relationships. Thus set theory — until then an area of higher mathematics that was taught at university – was declared the foundation of mathematics as a whole and of maths teaching. An American, Professor Dienes, designed 'logic blocks' as a basis for practical exercises.

The *second* complaint was that the teaching fixed arithmetic on the *decimal system*, which, it was said, was only *one* possibility among many — and — from a mathematical point of view — quite arbitrarily chosen. The use of the binary system in computer technology showed that it was necessary to loosen the hold of the decimal system over arithmetic.

Thirdly, it was at about the same time that the first electronic pocket calculators appeared on the market. That made it look as if modern people no longer needed to be able to do sums, since the machine could do everything more quickly and more reliably. The task of mathematics, it was claimed, was to teach pupils to understand arithmetical processes, but the actual calculations could happily be left to the machine.

These arguments were accepted in Switzerland, even though there was no comparison between the quality of teaching here and that in American public schools. Apart from that, I question all three arguments from a psychological and an educational point of view.

Firstly, our minds need thought routines if they are to be creative at all. We have to learn how to make logical deductions. If pupils vary this by doing thousands of sums, we are not fixing their thought processes, we are giving them the equipment to make future, more complex thought processes more manageable, even to make them possible. In mathematics the fundamental thought routines consist in dealing with number concepts and the basic operations. Naturally those elementary relationships between sets which can be demonstrated using the logic blocks are part of this. But it is certainly wrong to maintain that this abstract approach will enable a child to solve more concrete arithmetical problems.

Secondly, from the point of view of psychology, the decimal system is not an arbitrary product but is derived from our ten fingers. Relating it to this physical reality is, from a psychological and educational point of view, elementary in the truest sense of the word. An abstract concept is anchored in our own body and any other system depends on the basic idea and the linguistic conventions of the decimal system for us to be able to understand it. Children need an inner yardstick to help them find their way into another numerical system. In addition to that, in any area of learning children need to establish fixed points from which they can extend their knowledge and skills. This is especially important for less gifted children. Otherwise all we will do is produce failures.

Thirdly, the calculator cannot replace 'mental arithmetic', for without a clear conception of numbers we will be unable to interpret the mechanically produced series of figures adequately as numbers and values. Beyond that, there are many calculations we need to do in the course of our everyday lives for which we cannot constantly be taking out our pocket calculator. And finally, mental arithmetic serves the more general purpose — in Pestalozzi's system — of helping to develop our faculties: our imagination, our ability to store abstract material for short periods, our ability to deal in our minds with abstractions, our ability to concentrate. Education is not about producing results as quickly as possible, it is about our thought processes as such, for it is only by thinking that we can develop our ability to think.

During the last couple of decades the influence of the ideological positions that dominated the sixties and seventies has gradually receded. What it has left behind, in my opinion, are three relics which are partly responsible for our school leavers' unsatisfactory arithmetical skills:

No longer requiring the individual steps to be expressed in language. When a pupil is given the problem of calculating the price of 7 kilos of goods, for which he has been given the price for 5 kilos, he needs to recognise the proportion 7:5 and to apply that to the price. If you insist, it can be expressed as a formula, but no one can get round the fact that you have to divide the price of the goods by 5 and multiply the result by 7. Leaving aside simple addition and subtraction, this is the basic model for most mathematical problems faced by people today in their private life and in their work. It is in the true sense of the word an *elementary* calculation. Despite that, experience shows that many people find it almost impossible to solve such problems, or at least can only do so with great difficulty, especially when the actual figures are a little more complex.

It would help if educational theory did not reject a method simply because it had been used for decades, even centuries. Until the above-mentioned revolution in the teaching of mathematics in the 1970s, in Switzerland, the land of Pestalozzi, it was customary — and especially helpful for mathematically less gifted children — for pupils to be trained to express the individual steps of this type of calculation in language:

* 5 kilograms of potatoes cost 9 francs
* 1 kilogram of potatoes costs 9 francs ÷ 5 = 1.80 francs
* 7 kilograms of potatoes cost 7 x 1.80 francs = 12.60 francs

The almost complete elimination of the linguistic expression of calculations does a disservice to children. Everything that is done in mathematics at this level should be expressible in language. Then pupils' thinking will be founded on clear conceptions. The three brief sentences in the example given represent the stages of a correct logical deduction.

The abolition of the distinction between measuring and partitioning in division. That is all very well for a mathematician, who thinks in purely abstract terms and does not visualise anything specific under the factors or the operator 'times', but for pupils, who first of all have to understand how it all works, not distinguishing between the two aspects is fatal. Every calculation and every operator must be based on an *action* that is comprehensible in physical terms and can be visualised. Whatever the 'new mathematicians' may say, cutting a length of three metres up into sixty equal parts, for example, is not the same as ascertaining how many units of sixty centimetres are contained in the three metres. This obvious fact needs an equivalent in the mathematical representation, including the linguistic formulation: in the first case we are partitioning, in the second measuring. That is, the division operator has two meanings which must be clearly distinguished logically and linguistically.

Similarly, the multiplication operator must have a meaning which can be carried out as action and as visualisation. This is only possible when the first factor is the multiplier: if there are seven rulers, each of thirty centimetres, on the table, then the multiplicand (which is always the second factor) is repeated seven times. Whatever the commutative law might say, for beginners, who need to be able to base their thinking on actions and visualisations, the sum 'thirty times seven centimetres' is simply wrong, even though it produces the right result.

Pupils must also learn to see that when they are dividing they are inverting an actual or conceivable multiplication. Then they can say: If I make the *first* factor (the multiplier) the divisor, I am *partitioning* and the result is the multiplicand. If, on the other hand, I make the *second* factor (the multiplicand) the divisor, I am *measuring* and the result is the multiplier. Not distinguishing between measuring and partitioning has *not* led to more mathematical thinking, but to unclear dealing with numbers that is not founded on visualisation. It is the children who suffer, above all the weaker ones, who particularly need to start out from concrete actions and visualisation.

The general devaluation of arithmetic, of working things out in one's head and learning tables off by heart. This is a result of the extension of the range and variety of mathematical subject matter and exercises. There is no doubt that this makes mathematics lessons more varied and interesting — especially for the more gifted pupils. Unfortunately it simply leaves too little time to practise the elementary skills. We are therefore tempted to ask, would less not perhaps be more?

At the foundation stages of arithmetic I have had excellent results with the Cuisenaire method. This proceeds *analytically* rather than *synthetically*. The starting point is never a precise question, such as 'What does six times seven make?' but a result: 'What are all the things that make forty-two?' Central to it is the observation of numbers. The synthetic procedure only allows *one* solution to a problem, while with the analytical method the problem is left *completely open*. That means there are hardly any limits set on the pupils' freedom and creativity and that particularly increases their motivation. It also means that pupils of different ability can work at their own level, without the threat of failure.

I would prefer not to have to tell a critic of our school system what I quoted at the beginning of this chapter: 'True, they can't do sums, but they can think.' I would like to be able to say, 'Since they have learnt to think, they can also do sums.'